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Supersymmetric Noncommutative QED and Lorentz Violation

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Abstract

We consider Lorentz-violating operators induced at the loop level in softly-broken supersymmetric noncommutative QED. Dangerous operators forbidden in the supersymmetric limit are generated via finite corrections, with the scale of supersymmetry breaking serving as a gauge-invariant regulator. We compare the most dangerous loop effects to those obtained in noncommutative theories truncated by a momentum-space cutoff, and find significantly improved bounds.

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I. INTRODUCTION

The idea that spacetime may be modified in nontrivial ways at distance scales that are accessible at high energy colliders has led to recent interest in the phenomenology of noncommutative field theories [1, 2, 3, 4, 5, 6, 7, 8, 9]. Such theories are known to arise in string theory [10] and to have interesting properties [11]. The phenomenology of these theories is determined by a real, antisymmetric matrix θ which defines the fundamental spacetime commutation relation:

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu} . \quad (1.1)$$

Notice that the coordinate x^μ has been promoted to an operator \hat{x}^μ . Field theories on a noncommutative space can be constructed in terms of fields that are functions of commuting spacetime coordinates provided that ordinary multiplication is promoted to star multiplication. The star product corresponding to Eq. (1.1) is given by

$$(f \star g)(x) = f(x) \exp \left[\frac{i}{2} \overleftarrow{\partial}_\mu \theta^{\mu\nu} \overrightarrow{\partial}_\nu \right] g(x) , \quad (1.2)$$

for any two functions f and g , the well-known Moyal-Weyl result. A field theory action is then of the form

$$S = \int d^4x \mathcal{L}(\phi(x), \partial_\mu \phi(x))_\star , \quad (1.3)$$

where the \star subscript indicates that all multiplications between fields are defined as in Eq. (1.2). The specific form of \mathcal{L} is fixed by the usual requirements of invariance under the local and global symmetries of the theory.

By far, the largest number of studies in noncommutative phenomenology have been directed toward noncommutative QED (NCQED), both in four [1] or more [7] dimensions. The field strength tensor in NCQED is given by

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ie[A_\mu \star A_\nu] , \quad (1.4)$$

indicating the existence of three- and four-photon vertices. The observable effects of these new interactions have been considered in collider studies by a number of authors [1].

On the other hand, low-energy non-accelerator experiments may provide much more stringent [3], if not insurmountable [4, 5, 6], bounds on the size of θ . The most notable phenomenological feature of canonical noncommutative field theories is the violation of Lorentz

invariance following from Eq. (1.1). Both θ^{i0} and $\epsilon^{ijk}\theta_{jk}$ are fixed three-vectors that define preferred directions in a given Lorentz frame. Anisimov, Banks, Dine and Graesser [5] have pointed out that operators of lower mass dimension are generated via loop effects in noncommutative theories and these operators are severely constrained by low-energy searches for the violation of Lorentz invariance [12]. If the loop integrals are evaluated without a momentum space cutoff, the most dangerous Lorentz-violating operators receive contributions that are independent of the scale of θ . This is due to powers of θ^{-1} that appear after integration due to the peculiar momentum dependence of the noncommutative vertices. Experimental bounds in this case cannot be evaded by raising the scale of noncommutativity, and the underlying theory is excluded [5]. On the other hand, if a momentum-space cutoff Λ is used, for example, to take into account a change in the physics at a low Planck scale, then the Lorentz-violating effects depend on the scale of noncommutativity through the product $\theta\Lambda^2$, which can be bounded. In NCQED, consideration of the most dangerous operator

$$\mathcal{O}_1 = m_e \theta^{\mu\nu} \bar{\psi} \sigma_{\mu\nu} \psi \quad , \quad (1.5)$$

where ψ represents the electron field and m_e the electron mass, leads to the bound $\theta\Lambda^2 < 10^{-19}$ [5]; even tighter bounds have been shown to arise in noncommutative QCD [6]. These results suggest that if Lorentz-violating noncommutativity is realized in nature, the size of θ is much smaller than one would expect from naive dimensional arguments.

It is natural to question the reliability of bounds obtained via regulating a gauge theory with a hard, ultraviolet cutoff. Such a cutoff violates the gauge invariance of the theory, and is not defined precisely in terms of physical quantities. It is the purpose of this Letter to investigate whether the phenomenological conclusions described above are altered substantially when one employs a cutoff that is both physical and preserves gauge invariance.

We therefore focus on softly-broken supersymmetric NCQED. In the supersymmetric limit [13], one may show that the most dangerous Lorentz-violating operator in Eq. (1.5) is forbidden: there is no way to construct an F or D term using $\theta^{\mu\nu}$, superfields, and derivatives, that reduces to the desired Dirac and Lorentz structure. When supersymmetry is softly broken by giving the superpartners a common mass M , the dangerous operator in Eq. (1.5) is again generated; however, supersymmetric cancellations eliminate contributions from the ultraviolet part of the loop integrals. Thus, M serves as an effective cutoff that

preserves the gauge invariance of the theory. In the next section we adopt this framework in computing the operator in Eq. (1.5), which is generated at the two-loop level. The dependence on θM^2 differs from what one would expect given a hard cutoff, and leads to a stronger bound on the size of θ . We then comment upon Lorentz-violating corrections to the photon propagator and summarize our conclusions.

II. A DANGEROUS OPERATOR

Here we isolate the two-loop contribution to the operator in Eq. (1.5). In ordinary NCQED, there is no one-loop diagram that contributes to \mathcal{O}_1 . Two-loop diagrams that contribute are shown in Figs. 1 and 2.

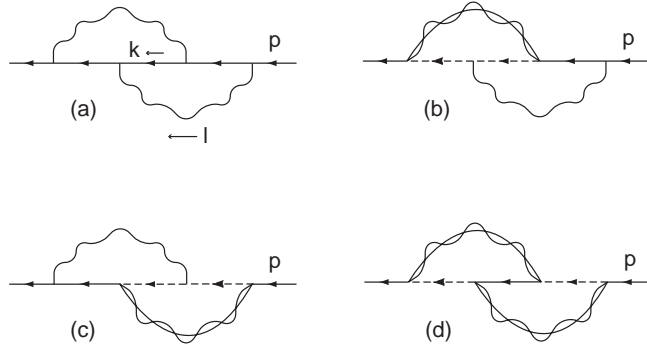


FIG. 1: Two-loop diagrams with two gauge multiplet propagators. Solid lines represent electrons, wavy lines represent photons, wavy lines with a solid core represent photinos, and dashed lines represent selectrons.

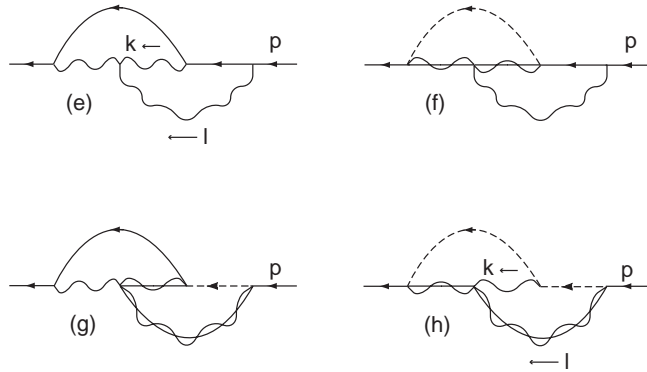


FIG. 2: Two-loop diagrams with three gauge multiplet propagators.

We will extract the terms proportional to $\sigma_{\mu\nu}$ and work on shell (*i.e.*, we evaluate the diagrams between spinors $\bar{u}(p)$ and $u(p)$ and use $\not{p}u(p) = m_e u(p)$.) For each of the 8 diagrams, the $\sigma_{\mu\nu}$ terms are proportional to the electron mass m_e . After extracting the overall electron mass factor, we set the electron mass and momentum p to zero in the integrals as a simplifying assumption. This leads to corrections in the final result that are wholly negligible as far as our numerical analysis is concerned.

Each of the diagrams with only electrons and photons, (a) and (e), give identical results, and the sum of the two is

$$\mathcal{M}_a + \mathcal{M}_e = 24im_e e^4 \int (dk)(dl) \frac{e^{il \cdot \theta \cdot k} \sigma_{\mu\nu} k^\mu l^\nu}{k^2 l^4 (k+l)^4}, \quad (2.1)$$

where $(dk) \equiv d^4k/(2\pi)^4$ and $l \cdot \theta \cdot k \equiv l_\mu \theta^{\mu\nu} k_\nu$. The result (using techniques shown below) is

$$\mathcal{M}_a + \mathcal{M}_e = \frac{1}{8} m_e \alpha^2 \frac{\sigma_{\mu\nu} \theta^{\mu\nu}}{\sqrt{(-1/2) \text{Tr } \theta^2}}. \quad (2.2)$$

Now consider the diagrams with superpartners. The four diagrams with three superpartner propagators all give the same result, at least to the operator $\sigma_{\mu\nu} \theta^{\mu\nu}$, and similarly for the two diagrams with four superpartner propagators. We will give some detail of how the diagrams are evaluated. Using diagram (h), as an example, we have

$$\mathcal{M}_h = -4im_e e^4 \int (dk)(dl) \frac{e^{il \cdot \theta \cdot k} \sigma_{\mu\nu} k^\mu l^\nu}{k^2 (l^2 - M^2)^2 ((k+l)^2 - M^2)^2}, \quad (2.3)$$

where M is a common superpartner mass. We can combine denominators using a Feynman parameter, and shift one of the integration momenta to obtain

$$\mathcal{M}_h = -24im_e e^4 \int_0^1 dx x(1-x) \int (dk)(dl) \frac{e^{il \cdot \theta \cdot k} \sigma_{\mu\nu} k^\mu l^\nu}{k^2 [l^2 + x(1-x)k^2 - M^2]^4}. \quad (2.4)$$

Say that only $\theta_{12} = -\theta_{21} \equiv \theta \neq 0$. Then

$$\mathcal{M}_h = 4m_e e^4 \sigma_{12} \frac{\partial}{\partial \theta} J_h, \quad (2.5)$$

where after rescaling k and Euclideanizing, we have

$$J_h = 6 \int_0^1 dx \int (dk)(dl) \frac{e^{i(l_1 k_2 - l_2 k_1) \theta / \sqrt{x(1-x)}}}{k^2 [l^2 + k^2 + M^2]^4}. \quad (2.6)$$

Now the $dl_0 dl_3$ integrals can be done. After combining the remaining denominators using another Feynman parameter and rescaling the remaining components of l , we get

$$J_h = \frac{3}{8\pi^3} \int_0^1 dx \int_0^1 dy y \int (dk) dl_1 dl_2 \frac{e^{i(l_1 k_2 - l_2 k_1) \theta / \sqrt{yx(1-x)}}}{[k^2 + l_1^2 + l_2^2 + yM^2]^4}. \quad (2.7)$$

Now do the $dk_0 dk_3$ integrals, and put the denominators into the exponential using a Schwinger parameter. After one more rescaling of the remaining momenta, we have

$$\begin{aligned}
J_h &= \frac{1}{256\pi^6} \int_0^1 dx \int_0^1 dy y \int_0^\infty dz \int dk_1 dk_2 dl_1 dl_2 \\
&\times e^{-yzM^2 - k_1^2 - k_2^2 - l_1^2 - l_2^2 + i(l_1 k_2 - l_2 k_1)\theta / (z\sqrt{yx(1-x)})} \\
&= \frac{1}{256\pi^4} \int_0^1 dx \int_0^1 dy y \int_0^\infty dz \frac{4z^2 yx(1-x)}{4z^2 yx(1-x) + \theta^2} e^{-yzM^2} .
\end{aligned} \tag{2.8}$$

Thus,

$$\mathcal{M}_h = -\frac{m_e \alpha^2}{2\pi^2} \sigma_{12} \int_0^1 dx \int_0^1 dy y \int_0^\infty dz \frac{4z^2 yx(1-x)\theta}{(4z^2 yx(1-x) + \theta^2)^2} e^{-yzM^2} . \tag{2.9}$$

The end result for \mathcal{M}_f is a similar expression, but with the front integrals reading

$$\int_0^1 dx x \int_0^1 dy (1-y) \dots . \tag{2.10}$$

Noting that the rest of the integrand is symmetric under inversion about $x = 1/2$, we can replace “ x ” in the line above by “ $(x - 1/2) + 1/2$,” and keep only the “ $1/2$.” The 6 graphs involving superpartners sum to

$$4\mathcal{M}_f + 2\mathcal{M}_h = -\frac{m_e \alpha^2}{\pi^2} \sigma_{12} \int_0^1 dx \int_0^1 dy \int_0^\infty dz \frac{4z^2 yx(1-x)\theta}{(4z^2 yx(1-x) + \theta^2)^2} e^{-yzM^2} . \tag{2.11}$$

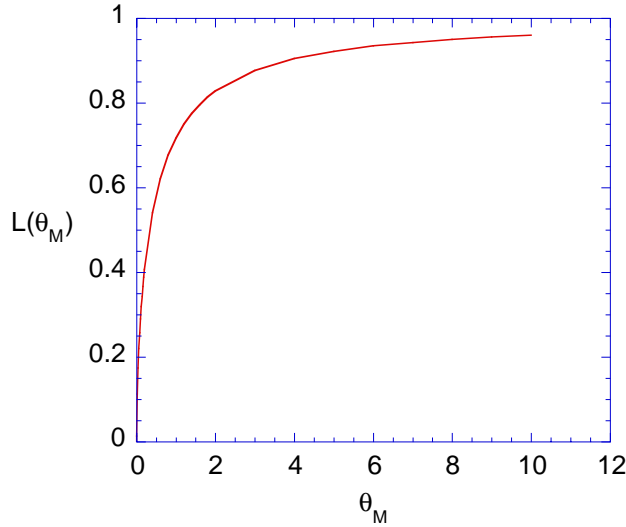


FIG. 3: The function $L(\theta_M)$.

In the supersymmetric limit $M \rightarrow 0$, the integrals can be done exactly. For general M , it is convenient to rescale z ,

$$4\mathcal{M}_f + 2\mathcal{M}_h = -\frac{m_e \alpha^2}{\pi^2} \frac{\sigma_{12}}{(\theta M^2)^3} \int_0^1 dx \int_0^1 dy \int_0^\infty dz \frac{4z^2 x(1-x)}{(4z^2 x(1-x)(\theta M^2)^{-2} + y)^2} e^{-z} . \tag{2.12}$$

The y and x integrals can both be done, and the full result for the two-loop contributions to \mathcal{O}_1 becomes

$$\mathcal{M} = \sum_{I=a}^g \mathcal{M}_I = \frac{1}{8} m_e \alpha^2 \frac{\sigma_{\mu\nu} \theta^{\mu\nu}}{\sqrt{(-1/2) \text{Tr } \theta^2}} L(\theta_M) , \quad (2.13)$$

where $\theta_M \equiv M^2 \sqrt{(-1/2) \text{Tr } \theta^2}$ and

$$L(\theta_M) = 1 - \frac{2\theta_M}{\pi^2} \int_0^\infty \frac{dz}{z \sqrt{z^2 + \theta_M^2}} e^{-z} \ln \frac{\sqrt{z^2 + \theta_M^2} + z}{\sqrt{z^2 + \theta_M^2} - z} . \quad (2.14)$$

The z integral can be computed analytically, but the answer is not enlightening and we do not show it. Function L satisfies $L(0) = 0$ and $L(\infty) = 1$ and is shown in Fig. 3.

For the choice $\theta^{0i} = 0$ [14], $\theta^{\mu\nu}$ defines a 3-vector in a fixed direction \hat{n} (where \hat{n} is a unit vector) and the result (2.13) can be written as a effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} R_\infty L(\theta_M) \vec{\sigma} \cdot \hat{n} , \quad (2.15)$$

where $R_\infty \equiv m_e \alpha^2 / 2 = 13.6$ eV. Searches for such a term in magnetic systems [15] show that matrix elements of \mathcal{L}_{eff} are below 10^{-19} eV. Doing without any cutoff, $L \rightarrow 1$, is impossible. One must get a severe suppression from $L(\theta_M)$, requiring $\theta_M \equiv M^2 / \Lambda_{NC}^2 \ll 1$. The slope of L near the origin is infinite, meaning that $L(\theta_M)$ has a nonanalytic behavior for $\theta_M \rightarrow 0$. Numerical evaluations suggest

$$L(\theta_M) \approx 3 (\theta_M)^{0.78} \quad (2.16)$$

for small argument. From this we estimate $\theta_M \lesssim 10^{-26}$ or $\Lambda_{NC} \gtrsim 10^{13} M$.

III. ONE-LOOP EFFECTS

In this section, we briefly consider Lorentz-violating operators that are quadratic in the photon field. We focus on

$$(\theta_{\alpha\beta} F^{\alpha\beta})^2 , \quad (3.1)$$

which is generated at the one-loop level [5]; here we will evaluate the contribution to this operator in the softly-broken supersymmetric theory.

Three one-loop diagrams are relevant to this computation: a photon loop, a ghost loop, and a photino loop. Letting p represent the external photon momentum, and ℓ a loop

momentum, the relevant contributions to the photon self-energy from photino, photon and ghost loops are given respectively by

$$i\Delta\Pi^{\mu\nu} = -16e^2 I^{\mu\nu}(M) + 20e^2 I^{\mu\nu}(0) - 4e^2 I^{\mu\nu}(0) , \quad (3.2)$$

where

$$I^{\mu\nu} = \int (d\ell) \frac{\ell^\mu \ell^\nu \sin^2[\frac{1}{2}\ell \cdot \theta \cdot p]}{[\ell^2 - M^2][(p + \ell)^2 - M^2]} . \quad (3.3)$$

Again, M represents the photino mass. In the supersymmetric limit, $\Delta\Pi^{\mu\nu}$ vanishes.

In terms of complex exponentials, Eq. (3.3) is similar in form to the integrals discussed in Section 2. In particular, one may evaluate the $\mu = \nu = 1$ element of $I^{\mu\nu}$, assuming that $\theta^{0i} = 0$ [14], and that the photon is on-shell and propagates in the 3 direction. The intermediate steps are similar to those discussed earlier in the two-loop example, so we do not present them. The correct Lorentz structure of the final result may be inferred from the specific case. We find

$$i\Delta\Pi^{\mu\nu} = \frac{ie^2}{8\pi^2} M^4 \theta^{\mu\alpha} p_\alpha \theta^{\nu\beta} p_\beta I(\xi) , \quad (3.4)$$

and

$$I(\xi) = \int_0^\infty dt \frac{(1 - e^{-t})}{t^3} \exp[-\xi/(4t)] , \quad (3.5)$$

where ξ represents the dimensionless combination $M^2(p_\mu \theta^{\mu\alpha} \theta_{\alpha\beta} p^\beta) \equiv M^2(p \cdot \theta \cdot \theta \cdot p)$. This last integral can be evaluated analytically, and expressed in terms of a modified Bessel function:

$$I(\xi) = \frac{16}{\xi^2} \left[1 - \frac{\xi}{2} K_2(\sqrt{\xi}) \right] . \quad (3.6)$$

To study the phenomenological consequences of this result, it is useful to consider the case in which ξ is small (for example, M and p of order the weak scale with noncommutativity at a high Planck scale). Notice that one may expand the quantity $\xi^2 I(\xi)$ as

$$\xi^2 I(\xi) = 4\xi + \left(\gamma - \frac{3}{4} - \ln(2) + \frac{1}{2} \ln(\xi) \right) \xi^2 + O(\xi^3) , \quad (3.7)$$

from which one may deduce

$$i\Delta\Pi^{\mu\nu} = \frac{ie^2 M^2}{2\pi^2} \left[\frac{\theta^{\mu\alpha} p_\alpha \theta^{\nu\beta} p_\beta}{p \cdot \theta \cdot \theta \cdot p} \right] + \frac{ie^2 M^4}{8\pi^2} \left(\gamma - \frac{3}{4} - \ln(2) \right) [\theta^{\mu\alpha} p_\alpha \theta^{\nu\beta} p_\beta] + \dots . \quad (3.8)$$

The result for off-shell photons may be obtained by replacing M^2 by $M^2 - z(1 - z)p^2$ and integrating the final result between $z = 0$ and 1. The second term in the expansion (3.8)

contributes directly to the operator of interest in Eq. (3.1). The dependence on M is the same as one would expect in a theory regulated by a momentum-space cutoff, and we therefore obtain the same bound given in Ref. [5], from limits on birefringent effects in light from distant galaxies, $\theta M^2 < 10^{-12}$. This is much weaker than the bound obtained in Section 2, $\theta M^2 \lesssim 10^{-26}$. The first term in Eq. (3.8) is peculiar in that it is independent of the scale of θ . In Ref. [16], it was suggested that such terms vanish in the case of softly-broken supersymmetry, in conflict with our explicit result. This term leads to a tachyonic photon [17] for certain polarizations, with a mass scale $\sim \sqrt{\alpha/\pi} M \sim 100$ GeV for $M \sim 1$ TeV, independent of the scale of θ . If this term is physical, it rules out canonical NCQED by itself, unless there is a further modification of the theory in the ultraviolet. However, it has also been suggested that terms like the first in Eq. (3.8) may be artifacts related to the use of Wess-Zumino gauge [18]. We therefore consider our result from Section 2 as a more conservative bound on the scale of noncommutativity.

IV. CONCLUSIONS

We have studied Lorentz-violating operators in softly-broken, supersymmetric noncommutative QED. In the limit where the superpartners decouple, we recover the nonsupersymmetric result that the most dangerous operators lead to conflict with experimental bounds [5]. In the opposite limit where supersymmetry is unbroken, the operators of interest are forbidden exactly. This statement follows because there is no supersymmetric way to write these operators; alternatively, one may see that the Feynman diagrams that contribute to the most dangerous operators cancel exactly. Including a soft mass M for the superpartners leads to non-zero values for the operator coefficients; the supersymmetry-breaking mass thus acts as a gauge-invariant regulator, allowing one to interpolate between these limits. In contrast to the results obtained by applying a simple momentum-space cutoff, the dependence on the superpartner mass M is not analytic as M goes to zero. The bound that follows from searches for Lorentz violation in magnetic systems [15] is seven orders of magnitude more severe, $\theta M^2 \lesssim 10^{-26}$, and places the scale of noncommutativity at or above the conventional supersymmetric grand unification scale for $M \sim 1$ TeV.

If nature uses noncommutative coordinates, it need not be done with a Lorentz-violating

implementation. One may take the results of the present investigation as motivation to pursue space-time noncommutativity in Lorentz-covariant ways [7, 19].

APPENDIX A: NONCOMMUTATIVE FEYNMAN RULES

The superpartner noncommutative QED Feynman rules that are needed for this paper are given in Fig. 4. The remaining Feynman rules can be found in the literature (e.g., [16]).

$$\begin{aligned}
& \text{Diagram 1: } \bar{\lambda} \text{ (wavy)} \xrightarrow{r, \alpha} \text{ vertex } \xrightarrow{q, \beta} \lambda \text{ (wavy)} \quad 2e \sin(\tfrac{1}{2} r \cdot \theta \cdot q) (\gamma_\mu)_{\alpha\beta} \\
& \text{Diagram 2: } \text{---} \xrightarrow{p'} \text{ vertex } \xrightarrow{p} \text{---} \quad -ie \exp(\tfrac{i}{2} p \cdot \theta \cdot p') (p' + p)_\mu \\
& \text{Diagram 3: } \bar{\psi} \text{ (solid)} \xrightarrow{p', \alpha} \text{ vertex } \xrightarrow{p} \text{---} \text{L} \quad \frac{e}{\sqrt{2}} (1 + \gamma_5)_{\alpha\beta} \exp(\tfrac{i}{2} p \cdot \theta \cdot p') \\
& \text{Diagram 4: } \text{---} \text{L} \xrightarrow{p'} \text{ vertex } \xrightarrow{p, \beta} \psi \text{ (solid)} \quad \frac{-e}{\sqrt{2}} (1 - \gamma_5)_{\alpha\beta} \exp(\tfrac{i}{2} p \cdot \theta \cdot p')
\end{aligned}$$

FIG. 4: Feynman rules for superpartners in noncommutative supersymmetric QED. The rules for the right-handed scalars can be obtained from the left-handed ones shown by $\gamma_5 \rightarrow -\gamma_5$. ψ and λ represent electrons and photinos, respectively. Our sign conventions are based on those of Ref. [20].

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